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BARTOSZ SOBCZYK

ESTIMATION OF FAILURE  
INITIATION IN LAMINATED  
COMPOSITES BY MEANS  
OF NONLINEAR SIX-FIELD  
SHELL THEORY AND FEM

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## LIST OF THE MOST IMPORTANT SYMBOLS AND NOTATIONS

$\underline{Q}(x)$	– tensor of mean rotary deformation of the shell cross sections
$T(x)$	– the structure tensor in the current configuration
$T_0(x)$	– the structure tensor in the reference (initial) configuration
$x$	– the position vector of initial configuration $M$
$\mathbf{e}$	– vector of shell strains
$\mathbf{s}$	– vector of shell stress and couple resultants
$\mathbf{C}$	– constitutive matrix
$\varphi^{(k)}$	– angle of fibre orientation in $k^{\text{th}}$ lamina
$(a^{(k)}, b^{(k)}, c^{(k)})$	– orthogonal material coordinate system of $k^{\text{th}}$ lamina
$\tilde{\mathbf{T}}_k$	– transformation matrix
$\mathbf{A}_{4 \times 4}$	– extensional stiffness matrix
$\mathbf{B}_{4 \times 4}$	– extension – bending coupling matrix
$\mathbf{D}_{4 \times 4}$	– bending stiffness matrix
$\mathbf{S}_{2 \times 2}$	– transverse shear stiffness matrix
$\mathbf{G}_{2 \times 2}$	– drilling resultants stiffness matrix
$\sigma_{aa}$	– normal stress in the 1 <sup>st</sup> material direction
$\sigma_{bb}$	– normal stress in the 2 <sup>nd</sup> material direction
$\sigma_{ab}$	– in-plane shear stress in the 2 <sup>nd</sup> material direction
$\sigma_{ba}$	– in-plane shear stress in the 1 <sup>st</sup> material direction
$\sigma_a$	– transverse shear stress on the a-c plane
$\sigma_b$	– transverse shear stress on the b-c plane
$E_a$	– elastic modulus of lamina in the 1 <sup>st</sup> material direction
$E_b$	– elastic modulus of lamina in the 2 <sup>nd</sup> material direction
$G_{ab}$	– in-plane shear modulus of lamina
$G_{ac}$	– transverse shear modulus on lamina a-c plane
$G_{bc}$	– transverse shear modulus on lamina b-c plane
$\nu_{ab}$	– major Poisson's ratio of lamina
$X_t$	– absolute value of lamina tensile strength in the 1 <sup>st</sup> material direction
$X_c$	– absolute value of lamina compressive strength in the 1 <sup>st</sup> material direction
$Y_t$	– absolute value of lamina tensile strength in the 2 <sup>nd</sup> material direction
$Y_c$	– absolute value of lamina compressive strength in the 2 <sup>nd</sup> material direction
$S_f$	– lamina in-plane shear strength
$S_t$	– lamina transverse shear strength
$\sigma_{nn}$	– normal stress component acting on lamina failure plane
$\sigma_{nl}$	– shear stress component acting on lamina failure plane in the fibre direction
$\sigma_{nt}$	– shear stress component acting on lamina failure plane perpendicular to $\sigma_{nl}$
$\theta_f$	– lamina failure plane inclination angle
$\eta_{w1}$	– Puck's criterion weakening factor